DEPARTMENT OF MATHEMATICS

Problem of the Month

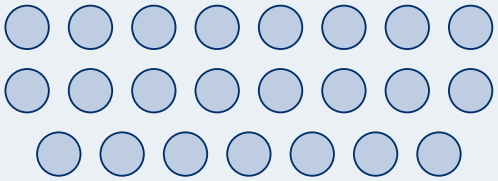
The Last Token

PennState Behrend

Three LionCash prizes | **Open to all Behrend undergraduates**

The game

Two players take turns removing tokens from one pile. On your turn, you must take 1, 2, 3, or 4 tokens. You cannot take more tokens than are left in the pile. The player that takes the last token loses the game.



Your challenge

1. Start with 23 tokens. Can you guarantee a win if you go first? If so, describe your first move and what you would do on later turns. Explain why your strategy works no matter how your opponent plays.

2. Start with any positive number of tokens. For which pile sizes can the second player guarantee a win? Describe all such sizes and explain why your list is complete.

How to enter

Contest rules

Submit during the last week of the month.

Work independently. You may play practice rounds with a friend. Write your solution without AI assistance or outside solutions.

Solution check. You may be invited to meet with a professor to explain your reasoning and try a small variation.

Win LionCash. The first three students with complete and correct solutions who successfully explain their reasoning earn LionCash prizes.

Try a round with a friend. Can you find a winning strategy?