Homework 02

Math 482: Mathematical Methods of Operations Research (Spring 2026)
Weeks 2–3 (Jan 21–Jan 30, 2026)

Relevant topics: Simplex Method, Auxiliary Method, Infeasibility and Unboundedness.
Due: Friday February 6, 2026.
Instructions: Show your work clearly; when working through the simplex method, clearly label the basis, non-basic variables, and basic solution. Submit your work in class on the due date.

I. Consider the primal LP shown below.

$$\begin{aligned} \text{maximize} \quad & z = 4x_1 + 5x_2 \\ \text{subject to} \quad & 2x_1 - 3x_2 \le -1, \\ & 4x_1 + x_2 \le 6, \\ & x_1 + x_2 \le 5, \\ & x_i \ge 0,\ \forall i \in \{1, 2\} \end{aligned}$$

a. Draw the feasible region of the given primal LP.
b. Write the initial primal tableau T, basis β, and non-basic variables π. Show that the tableau T is infeasible.
c. Write the auxiliary LP.
d. Draw the feasible region of the auxiliary LP.
e. Use the auxiliary method to determine a feasible primal tableau.
f. Plot the points from the auxiliary method on the drawing from d.
g. Use the simplex method to find the optimal primal tableau.
h. Plot the points from the simplex method on the drawing from a.

II. Consider the following primal LP

$$\begin{aligned} \text{maximize} \quad & z = x_1 + x_2 \\ \text{subject to} \quad & -x_1 + x_2 \le 2, \\ & x_2 \le 4, \\ & x_i \ge 0,\ \forall i \in \{1, 2\}. \end{aligned}$$

(a) Draw the feasible region and indicate a ray that proves the feasible region is unbounded.
(b) Write the initial tableau. Using the least subscript rule, identify the entering variable. Explain why the minimum ratio test fails (i.e., there is no leaving variable) and conclude that the LP is unbounded.
(c) Give an explicit *unbounded direction* $d \in \mathbb{R}^2$ such that $x + td$ is feasible for all $t \ge 0$ (for some feasible x) and such that $z(x + td) \to \infty$ as $t \to \infty$.