Homework 01

Math 482: Mathematical Methods of Operations Research (Spring 2026)
Week 1 (Jan 12–Jan 16, 2026)

Relevant topics: Linear Programming, Standard Form, Dual, Weak Duality Theorem
Due: Friday Jan 23, 2026.
Instructions: Show your work clearly. Submit your work in class on the due date.

I. Consider the following primal LP

$$\begin{aligned} \text{maximize} \quad & z = 3x_1 + 4x_2 \\ \text{subject to} \quad & 2x_1 - 3x_2 \leq 3, \\ & 4x_1 + x_2 \leq 6, \\ & x_1 + x_2 \leq 5, \\ & x_i \geq 0, \ \forall i \in \{1, 2\} \end{aligned}$$

a. Write the matrices A, $\mathbf{b}$, and $\mathbf{c}$ that correspond to the primal problem.

b. Write the dual problem.

c. Let w denote the objective function of the dual problem. Prove for this particular problem that $z \leq w$. Explain each step.

II. Consider the following primal LP

$$\begin{aligned} \text{maximize} \quad & z = 5x_1 + 3x_2 \\ \text{subject to} \quad & x_1 + 2x_2 \leq 14, \\ & 3x_1 - 2x_2 \leq 18, \\ & -x_1 + 2x_2 \leq 10, \\ & x_i \geq 0, \ \forall i \in \{1, 2\} \end{aligned}$$

a. Graph the system of constraints of the primal problem.

b. Add the following lines to your graph.

- $z = 12$, i.e., $5x_1 + 3x_2 = 12$
- $z = 24$
- $z = 36$

c. Add the following points to your graph.

- $\mathbf{x} = (2, 3)^T$
- $\mathbf{x} = (4, 6)^T$
- $\mathbf{x} = (8, 3)^T$

d. Find $\mathbf{x}^*$ and z^*.

e. Write the dual problem.

f. Find $\mathbf{y}^*$ and w^*.

III. Consider the following primal LP.

$$\begin{aligned} \text{maximize} \quad & z = -2x_2 \\ \text{subject to} \quad & x_1 + 4x_2 - x_3 \leq 1, \\ & -2x_1 - 3x_2 + x_3 \leq -2, \\ & 4x_1 + x_2 - x_3 \leq 1, \\ & x_i \geq 0, \ \forall i \in \{1, 2, 3\} \end{aligned}$$

a. Write the dual problem.

b. Show that $\mathbf{y} = (2t, 3t, t)^T$ is a feasible solution to the dual problem for all $t \geq 0$.

c. Show that the dual problem is unbounded and the primal problem is infeasible.