

# Uniform Continuity

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## 1 Exercises

- I. Show that  $f(x) = \frac{1}{x}$  is uniformly continuous on  $[2, \infty)$ .
- II. Find a continuous function  $f: S \rightarrow \mathbb{R}$  and a Cauchy sequence  $x: \mathbb{N} \rightarrow S$  such that  $f \circ x$  is a divergent sequence.
- III. Let  $f: S \rightarrow \mathbb{R}$  be uniformly continuous on  $S$  and suppose that  $x: \mathbb{N} \rightarrow \mathbb{R}$  is a Cauchy sequence. Prove that  $f \circ x$  is a Cauchy sequence.