

Subsequences Worksheet

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1 Exercises

I. Let $s = (1/n)_{n=1}^{\infty}$. For each subsequence below, identify the strictly increasing sequence $\sigma: \mathbb{N} \rightarrow \mathbb{N}$.

(a) $s \circ \sigma = (\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots)$

(b) $s \circ \sigma = (\frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \dots)$

(c) $s \circ \sigma = (\frac{1}{2}, \frac{1}{5}, \frac{1}{10}, \frac{1}{17}, \dots)$

II. Let $S \subseteq \mathbb{R}$ be non-empty. Prove that S is compact if and only if every sequence $s: \mathbb{N} \rightarrow S$ has a subsequence that converges to a point in S .