Homework 3

Real Analysis

Due October 10, 2025

Exercises

1. Classify each of the following sets as open, closed, neither, or both.
   (a) $A = (2, 5)$
   (b) $B = [2, 5]$
   (c) $C = [2, 5)$
   (d) $D = \mathbb{R}$
   (e) $E = \mathbb{Z}$
   (f) $F = \mathbb{Q}$
2. Use the open cover definition of compactness to prove that all finite sets are compact.
3. Let $S \subseteq \mathbb{R}$. Prove that if $x \in \mathbb{R}$ is an accumulation point of S, then every neighborhood of x contains infinitely many points of S.
4. Let $S \subseteq \mathbb{R}$ be bounded. Prove that if S contains infinitely many points, then there exists at least one point in $\mathbb{R}$ that is an accumulation point of S.
5. Let $S \subseteq \mathbb{R}$. Prove that $x \in S'$ if and only if there exists a sequence $s \colon \mathbb{N} \to \mathbb{R}$ such that $\operatorname{rng}(s) \subseteq S \setminus \{x\}$ and $\lim_{n\to\infty} s_n = x$.