

Homework 2

Real Analysis

Due September 17, 2025

Exercises

1. Consider the rational sequence $(a_n)_{n=1}^{\infty}$ defined as follows

$$a_1 = 2, \quad a_{n+1} = \frac{1}{2} \left(a_n + \frac{2}{a_n} \right), \quad \forall n \geq 1.$$

Prove that this sequence is decreasing and bounded below.

2. Prove that the sequence in problem 1 is Cauchy.
3. Prove the density of the rationals in the reals. That is for each $x, y \in \mathbb{R}$ such that $x < y$, prove that there is a $q \in \mathbb{Q}$ such that $x < q < y$.