Homework 1

Real Analysis

Due September 5, 2025

1 Exercises

1. Prove Theorem 4.1 from the Set Theory notes.

2. Observe that

$$1 = 1^2$$
$$1 + 3 = 2^2$$
$$1 + 3 + 5 = 3^2$$

   Determine a general formula and prove it using mathematical induction.

3. In Theorem 2.3 of the Natural Numbers notes, we proved the Well-Ordering property from the principle of mathematical induction. Now, prove the principle of mathematical induction from the Well-Ordering property. Hint: Use proof by contradiction.

4. Write a paragraph explaining what it means for the rational numbers to be an ordered field. Without providing a complete derivation, explain where the ordering and field properties come from.