

Continuous Functions Worksheet

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1 Exercises

- I. Let $S \subseteq \mathbb{R}$. Show that every point $c \in S$ is either an isolated point of S or an accumulation point of S .
- II. Define the modified Dirichlet function $f: (0, 1) \rightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} \frac{1}{n} & \text{if } x = m/n \text{ is a fully reduced rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

Show that f is continuous at each irrational in $(0, 1)$ and discontinuous at each rational.

- III. Suppose that $f: [a, b] \rightarrow \mathbb{R}$ is continuous and $f([a, b]) \subseteq \mathbb{Q}$. Prove that f is constant.