Trig Review

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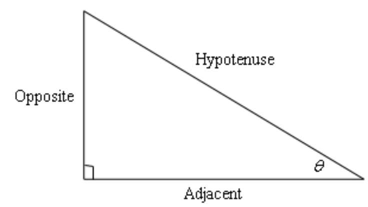
1 Trigonometric Functions

The six standard trig functions are defined over the right triangle

$$\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}} \quad \csc(\theta) = \frac{\text{Hypotenuse}}{\text{Opposite}}$$
$$\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}} \quad \sec(\theta) = \frac{\text{Hypotenuse}}{\text{Adjacent}}$$
$$\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}} \quad \cot(\theta) = \frac{\text{Adjacent}}{\text{Opposite}}$$



From these definitions, we can derive some important identities. To this end, let O denote the opposite side, A denote the adjacent side, and H the hypotenuse side of the right triangle. For example, consider the Pythagorean Identity:

$$\begin{aligned}\sin^2(\theta) + \cos^2(\theta) &= \left(\frac{O}{H}\right)^2 + \left(\frac{A}{H}\right)^2 \\ &= \frac{O^2 + A^2}{H^2} = \frac{H^2}{H^2} = 1.\end{aligned}$$

1.1 Unit Circle

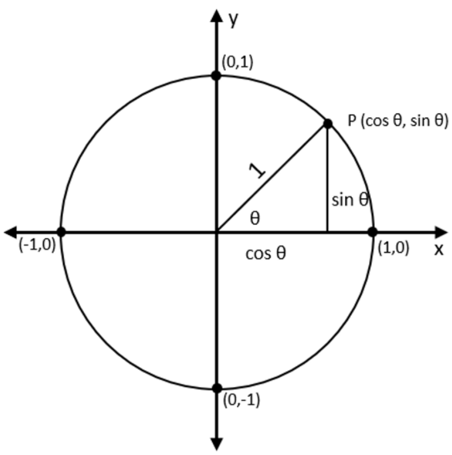
By embedding right triangles into the unit circle (circle of radius 1 centered at the origin), we are able to identify points on the circle using cosine and sine functions. For example, see Figure 1a where we have a right triangle with hypotenuse 1 embedded in the 1st quadrant of the unit circle such that the adjacent side lies on the x-axis and the opposite side is parallel to the y-axis. Hence, the points on the unit circle in the 1st quadrant can be identified by $(\cos(\theta), \sin(\theta))$ as defined on the right triangle.

We extend the definitions of $\cos(\theta)$ and $\sin(\theta)$ for all angles of θ using the points on the unit circle. For example, the sine (red) and cosine (blue) functions are shown over the domain $[-\pi, \pi]$ in Figure 1b.

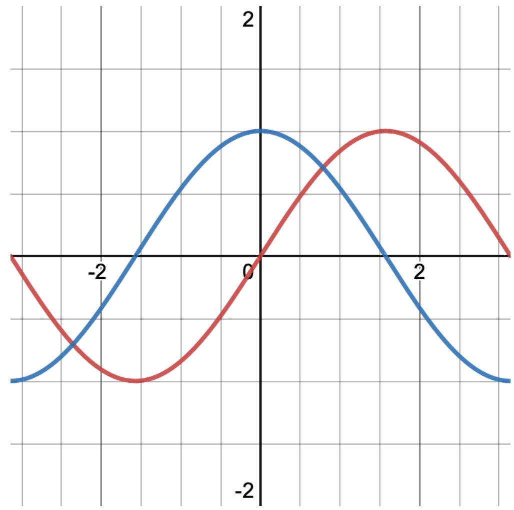
1.2 Inverse Trig Functions

The six standard trig functions map an angle θ to a ratio. The inverse trig functions map a ratio to an angle θ. For example,

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \Leftrightarrow \arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$
$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \Leftrightarrow \arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$$
$$\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}} \Leftrightarrow \arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$



(a) Unit circle with right triangle embedded inside the 1st quadrant



(b) Sine (red) and Cosine (blue) functions

Note that a function must be one-to-one on its domain in order for the inverse function to exist. Hence, we must restrict the domain of each standard trig function in order to have a well-defined inverse. For example, $\sin(x)$ is one-to-one on the domain $[-\frac{\pi}{2}, \frac{\pi}{2}]$. Moreover, over this domain, $\sin(x)$ has a range of $[-1, 1]$. Hence, the inverse $\arcsin(x)$ is well-defined on the domain $[-1, 1]$ with range $[-\frac{\pi}{2}, \frac{\pi}{2}]$, see Figure 2.

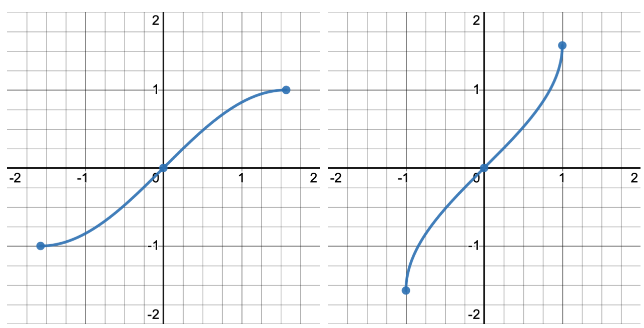


Figure 2: Sine (left) and ArcSine (right) functions