MATH-141 **Name:** _____
Fall 2025
Exam IV Worksheet
December 3, 2025 **Pledge:** _____

Each question topic and point value is recorded in the tables below. Note that this exam must be completed within the 50 minutes allotted. Also, you must work without any external resources, which includes no notes, calculator, nor any equivalent software. You must show an appropriate amount of work to justify your answer for each problem. If you run out of room for a given problem, you may continue your work on the back of the page. By writing your name and signing the pledge you are stating that you understand the rules outlining this exam.

Scoring Table

Question	Points	Score
1	10	
2	8	
3	8	
4	8	
5	10	
Total:	44	

Topics Table

Question	Topic
1	Conic Equations
2	Tangent Lines of Parametric Equations
3	Arc Length with Parametric Equations
4	Polar Coordinates
5	Area and Surface Area in Polar Coordinates

Reduction Formulas

$$\int \sin^n(x)dx = -\frac{1}{n}\cos(x)\sin^{n-1}(x) + \frac{n-1}{n}\int \sin^{n-2}(x)dx$$
$$\int \cos^n(x)dx = \frac{1}{n}\cos^{n-1}(x)\sin(x) + \frac{n-1}{n}\int \cos^{n-2}(x)dx$$
$$\int \tan^n(x)dx = \frac{1}{n-1}\tan^{n-1}(x) - \int \tan^{n-2}(x)dx$$
$$\int \sec^n(x)dx = \frac{1}{n-1}\sec^{n-2}(x)\tan(x) + \frac{n-2}{n-1}\int \sec^{n-2}(x)dx$$

1. (a) (5 points) Identify the center, foci, and vertices of the following conic

$$9x^2 - 36x - 4y^2 + 8y - 4 = 0.$$

What type of conic is it?

(b) (5 points) Write the equation of an ellipse that has foci $(3, 0)$ and $(-3, 0)$ with minor axis of length 8.

2. Consider the equation of the ellipse

$$\frac{(x-2)^2}{9} + \frac{(y+1)^2}{4} = 1.$$

(a) (2 points) Find a parameterization of the ellipse.

(b) (6 points) Find where the points on the ellipse have a tangent line with a slope of $-2/3$. What is the equation of the tangent line at these points?

3. Consider the curve formed by the parameterization

$$x(t) = 3t^2,\ y(t) = 2t^3,\ 0 \leq t \leq 1.$$

(a) (2 points) Sketch the graph of the curve.

(b) (6 points) Find the arc length of the curve.

4. (a) (2 points) Plot the rectangular coordinate $(-1, -1)$ and determine its polar form.

(b) (2 points) Plot the polar coordinate $(1, \pi/3)$ and determine its cartesian form.

(c) (4 points) Show that $r = 4\cos(\theta)$ represents a circle. What is the center and radius of this circle?

5. Consider the curve described by the polar equation

$$r = 1 - \sin(\theta),\ 0 \leq \theta \leq \frac{\pi}{2}.$$

(a) (2 points) Sketch the graph of the curve.

(b) (4 points) Find the area bounded by the curve and the x-axis.

(c) (4 points) Find the surface area of the solid formed by revolving the curve about the y-axis.