Math 140 Worksheet 3

Week 3 (through Wednesday): Continuity and the Intermediate Value Theorem

Instructions. Write clear solutions on your own paper. Show enough work to justify your answers. Upload a single PDF of your work to Canvas.

1. **(Continuity at a point)** Consider

$$F(x) = \begin{cases} x^2 - 1, & x < 2, \\ c, & x = 2, \\ 3x - 5, & x > 2. \end{cases}$$

   Find the value of c that makes F continuous at $x = 2$.

2. **(Where is it continuous?)** Let $r(x) = \dfrac{x^2 - 9}{x - 3}$.

   (a) Simplify $r(x)$ for $x \neq 3$.

   (b) Is r continuous at $x = 3$? Explain.

   (c) Modify the definition at $x = 3$ to create a continuous function on all real numbers.

3. **(IVT: existence of a root)** Let $p(x) = x^3 - 7x + 1$.

   (a) Show that $p(x)$ has a root in the interval $(2, 3)$.

   (b) Explain why the Intermediate Value Theorem applies.

4. **(IVT + approximation)** Consider $f(x) = \cos x - x$ on $[0, 1]$.

   (a) Use the Intermediate Value Theorem to show that f has a zero in $(0, 1)$.

   (b) Do one step of the bisection method starting with $[0, 1]$ to produce a smaller interval that still contains a zero.