

# Math 140 Worksheet 2 — Solution Key

## Week 2 (through Wednesday)

1. For  $x \neq 2$ ,  $\frac{x^2 - 4}{x - 2} = \frac{(x - 2)(x + 2)}{x - 2} = x + 2$ , so compute  $x + 2$ :

$x$	1.9	1.99	2.01	2.1	2.001
$\frac{x^2 - 4}{x - 2}$	3.9	3.99	4.01	4.1	4.001

Estimate: the values approach 4, so the limit is 4.

2. Factor:

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \lim_{x \rightarrow 2} \frac{(x - 2)(x + 2)}{x - 2} = \lim_{x \rightarrow 2} (x + 2) = 4.$$

3. (a)  $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (x + 2) = 3$ .  
 $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (3 - x) = 2$ .

(b) Since  $3 \neq 2$ , the two-sided limit  $\lim_{x \rightarrow 1} f(x)$  does not exist.

4. We need  $\lim_{x \rightarrow 1} f(x) = 3$  and continuity at  $x = 1$ . From the left:  $\lim_{x \rightarrow 1^-} (x^2 + b) = 1^2 + b = 1 + b$ . For the limit to equal 3, we require  $b = 2$ . From the right:  $\lim_{x \rightarrow 1^+} (ax + 2) = a + 2$ . For the limit to equal 3, we require  $a = 1$ .