Math 140 Final Exam Review Worksheet

Final Exam Review: Cumulative Practice

Instructions. This worksheet is for final exam preparation. It will not be collected. Work through the problems on your own paper, show enough work to justify your answers, and use the problems to identify topics that need more review.

This worksheet reviews algebra, limits, continuity, derivatives, applications of derivatives, integration, Riemann sums, the Fundamental Theorem of Calculus, and differential equations.

1. Complete the square and write the polynomial in vertex form. Then state the vertex.

$$p(x) = 3x^2 - 18x + 7.$$

2. Factor the denominator and find the partial fraction decomposition.

$$\frac{5x+1}{x^2-x-6}.$$

   Then state the domain of the original rational function.

3. Solve for x:

$$\ln(x-2) + \ln(x+1) = \ln(12).$$

4. Evaluate exactly:

$$\sin\left(\arctan\left(\frac{3}{4}\right)\right).$$

5. Simplify the expression and state the domain of the original expression:

$$\frac{x^2-9}{x^2+x-6}.$$

6. Evaluate the limit:

$$\lim_{x\to 2} \frac{x^2-4}{x-2}.$$

7. Evaluate the limit:
$$\lim_{x\to 0}\frac{\sqrt{1+5x}-1}{x}.$$

8. Consider the graph of a function f. Use the graph to evaluate:
$$\lim_{x\to 1^-} f(x), \qquad \lim_{x\to 1^+} f(x), \qquad \lim_{x\to 1} f(x), \qquad f(1).$$

9. Find a and b so that f is continuous at $x = 2$:
$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2}, & x < 2, \\ a, & x = 2, \\ b + \ln(x-1), & x > 2. \end{cases}$$

10. Use the limit definition of the derivative to find $f'(x)$ for
$$f(x) = 2x^2 - 3x.$$

11. Find $f'(x)$:
$$f(x) = x^5 - 4x^3 + 2e^x - 3\ln x + \sin x.$$

12. Differentiate:
$$g(x) = \frac{x^2\cos x}{x+1}.$$

13. Differentiate:
$$h(x) = \sqrt{1+e^{2x}}.$$

14. Use implicit differentiation to find $\dfrac{dy}{dx}$. Then find the equation of the tangent line at the given point.
$$x^2y + \ln(y) = x + 1, \qquad (0, e).$$

15. Let
$$y = \arctan(x).$$
Rewrite this as an equation involving $\tan(y)$, then use implicit differentiation to derive the derivative of $\arctan(x)$.

16. A 13-ft ladder rests against a vertical wall. The bottom of the ladder slides away from the wall at 2 ft/s. How fast is the top of the ladder sliding down the wall when the bottom of the ladder is 5 ft from the wall?

17. Let
$$f(x) = x^3 - 3x + 1.$$
Show that Rolle's Theorem applies on $[-2, 1]$, then find all values of c guaranteed by the theorem.

18. Let
$$f(x) = x^2$$
on the interval $[1, 3]$. Show that the Mean Value Theorem applies, then find all values of c guaranteed by the theorem.

19. For
$$f(x) = xe^{-x},$$
find the critical numbers, the intervals on which f is increasing or decreasing, the intervals on which f is concave up or concave down, and any inflection points.

20. A rectangle is placed under the curve
$$y = 16 - x^2$$
and above the x-axis, symmetric about the y-axis. Find the dimensions of the rectangle with maximum area.

21. Evaluate:
$$\int \left(5x^4 - \frac{2}{x} + 3\cos x + e^x\right)\, dx.$$

22. Evaluate:
$$\int 2x\sqrt{x^2 + 5}\, dx.$$

23. Evaluate using substitution. Change the bounds of integration.
$$\int_0^1 xe^{x^2+2}\, dx.$$

24. Evaluate:
$$\int \sin^3(x)\cos^2(x)\, dx.$$

25. Evaluate:

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx.$$

26. Evaluate:

$$\int \frac{x+3}{x^2+1}\,dx.$$

27. Solve the initial value problem:

$$\frac{dy}{dx} = x(1+y^2), \qquad y(0) = 0.$$

28. Solve the initial value problem:

$$y' + 2y = e^x, \qquad y(0) = 1.$$

29. Let $f(x) = x^2 + 1$ on $[0, 2]$. Write A_n^{right} as a summation and compute

$$\lim_{n\to\infty} A_n^{\text{right}}.$$

30. Consider

$$\int_a^b \frac{1}{x}\,dx, \qquad 0 < a < b.$$

Partition $[a, b]$ into n equal subintervals with endpoints

$$x_i = a + i\Delta x, \qquad \Delta x = \frac{b-a}{n}.$$

Find a choice of sample point $c_i \in [x_{i-1}, x_i]$ that makes the Riemann sum

$$\sum_{i=1}^{n} \frac{1}{c_i}\Delta x$$

telescope. Then explain what the sum becomes.