Calculus I Review I Worksheet

Math 140: Calculus with Analytic Geometry

Name: ______________________________ **Date:** ____________________

Directions. Show enough work to support each answer. Give exact values. No calculators or other electronic devices are allowed.

1. **Precalculus essentials.**

   Complete each task. Preserve all domain restrictions when simplifying.

   (a) For

   $$r(x) = \frac{x^2 - x - 6}{x^2 - 4x + 3},$$

   simplify $r(x)$, state its domain in interval notation, and identify the coordinates of any hole and the equation of any vertical asymptote.

   (b) Evaluate $\cos\left(\arctan \frac{3}{4}\right)$ exactly.

   (c) Solve $\ln(x-1) + \ln(x+1) = \ln(3x+1)$. Check the domain restrictions.

   (d) Solve $4^{x-1} = 8$ exactly.

2. **Limits and continuity from a graph.**

   Use the graph of $y = f(x)$ to answer the questions. If a limit does not exist, write DNE and explain why.

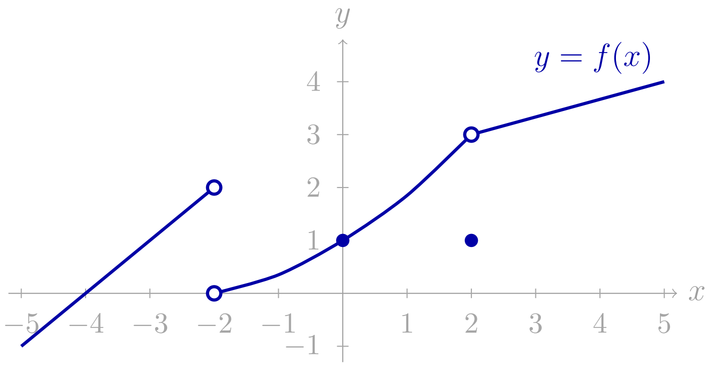

Figure 1: Graph of $y = f(x)$. The graph has separate pieces at $x = -2$ and $x = 2$; open and filled points indicate excluded and included values.

   (a) $\lim_{x \to -2^-} f(x) =$ ________________

   (b) $\lim_{x \to -2^+} f(x) =$ ________________

   (c) $\lim_{x \to -2} f(x) =$ ________________

   (d) $\lim_{x \to 2} f(x) =$ ________________

   (e) $f(2) =$ ________________

   (f) At each of $x = -2$ and $x = 2$, decide whether f is continuous. If not, classify the discontinuity as removable, jump, or infinite.

3. **Understanding the definition of a limit.**

   (a) Complete the following sentence: $\lim_{x \to c} f(x) = L$ means that $f(x)$ can be made ________________ to L by taking x ________________ to c (but not equal to c).

   (b) Decide whether each statement is true or false. If it is false, give a brief explanation or counterexample.

      i. If $f(2) = 1$, then $\lim_{x \to 2} f(x) = 1$.

      ii. If $\lim_{x \to 2^-} f(x) = 3$ and $\lim_{x \to 2^+} f(x) = 3$, then $\lim_{x \to 2} f(x) = 3$.

      iii. Changing the value of f at $x = 2$ can change $\lim_{x \to 2} f(x)$.

4. **Algebraic and transcendental limits.**

   Evaluate each limit. First state what direct substitution reveals, and then show the algebraic step, limit law, or continuity property that determines the limit.

   (a) $\displaystyle\lim_{x\to 3} \frac{x^2 - x - 6}{x^2 - 9}$

   (b) $\displaystyle\lim_{x\to 0} \frac{\sqrt{9+x} - 3}{x}$

   (c) $\displaystyle\lim_{x\to 0} \frac{\sin(4x)}{3x}$

   (d) $\displaystyle\lim_{x\to 1} \ln(x^2 + 3)$

5. **The Squeeze Theorem.**

   Use the Squeeze Theorem to evaluate

$$\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right).$$

   Begin with $-1 \leq \sin(1/x) \leq 1$, produce explicit lower and upper bounds for the given function, and verify that both bounding functions have the same limit.

6. **Continuity and piecewise-defined functions.**

   Let

$$g(x) = \begin{cases} \dfrac{x^2 - 1}{x - 1}, & x < 1, \\ b, & x = 1, \\ ax + 1, & x > 1. \end{cases}$$

   (a) Find the values of a and b that make g continuous at $x = 1$. Verify all three conditions in the definition of continuity at a point.

   (b) Suppose instead that $a = 3$ and $b = 2$. Classify the discontinuity at $x = 1$ and identify which continuity condition fails.