

Precalculus Review Quiz

Math 140: Calculus with Analytic Geometry

Answer Key

1. Rational expressions and domains.

Let

$$r(x) = \frac{x^2 - x - 6}{x^2 - 4x + 3}.$$

- (a) Factor the numerator and denominator and simplify $r(x)$.
- (b) State the domain of the *original* function in interval notation.
- (c) Identify any hole and any vertical asymptote of its graph.

Solution. Since

$$r(x) = \frac{(x-3)(x+2)}{(x-1)(x-3)} = \frac{x+2}{x-1}, \quad x \neq 1, 3,$$

the domain is $(-\infty, 1) \cup (1, 3) \cup (3, \infty)$. The canceled factor produces a hole at $(3, \frac{5}{2})$, and $x = 1$ is a vertical asymptote.

2. Polynomial equations.

Solve each equation over the real numbers.

- (a) $x^3 - 4x^2 - x + 4 = 0$
- (b) $2x^2 + 4x - 3 = 0$

Solution. Factoring by grouping gives

$$x^2(x-4) - 1(x-4) = (x-4)(x-1)(x+1),$$

so the solutions in part (a) are $x = -1, 1, 4$. For part (b), the quadratic formula gives

$$x = \frac{-4 \pm \sqrt{16 + 24}}{4} = \frac{-2 \pm \sqrt{10}}{2}.$$

3. Composition and domain.

Let

$$f(x) = \sqrt{x+1} \quad \text{and} \quad g(x) = \frac{1}{x-2}.$$

Find formulas for $(f \circ g)(x)$ and $(g \circ f)(x)$. State the domain of each composite function in interval notation.

Solution. We have

$$(f \circ g)(x) = \sqrt{\frac{1}{x-2} + 1} = \sqrt{\frac{x-1}{x-2}}.$$

The radicand must be nonnegative and $x \neq 2$. A sign chart gives the domain $(-\infty, 1] \cup (2, \infty)$. Also,

$$(g \circ f)(x) = \frac{1}{\sqrt{x+1}-2}.$$

Here $x \geq -1$, and the denominator is zero when $x = 3$. Thus the domain is $[-1, 3) \cup (3, \infty)$.

4. Inverse functions.

The function

$$h(x) = \frac{2x-3}{x+4}$$

has domain $\mathbb{R} \setminus \{-4\}$.

- (a) Find $h^{-1}(x)$.
- (b) State the domain of h^{-1} .
- (c) Verify your answer by showing that $h(h^{-1}(x)) = x$.

Solution. Solving $y = (2x-3)/(x+4)$ for x gives

$$yx + 4y = 2x - 3, \quad x(y-2) = -(3+4y), \quad x = \frac{3+4y}{2-y}.$$

Therefore $h^{-1}(x) = (4x+3)/(2-x)$, with domain $\mathbb{R} \setminus \{2\}$. Finally,

$$h(h^{-1}(x)) = \frac{2\left(\frac{4x+3}{2-x}\right) - 3}{\left(\frac{4x+3}{2-x}\right) + 4} = \frac{11x/(2-x)}{11/(2-x)} = x.$$

5. Exponential and logarithmic functions.

- (a) Solve $3^{2x-1} = 7$. Express your answer using logarithms.
- (b) Solve $\ln(x-1) + \ln(x+3) = \ln(5x+7)$, checking all domain restrictions.
- (c) Expand $\ln\left(\frac{x^3\sqrt{x+1}}{e^{2x}}\right)$, assuming $x > 0$.

Solution. Taking logarithms in part (a) gives $(2x-1)\ln 3 = \ln 7$, so

$$x = \frac{1}{2} \left(1 + \frac{\ln 7}{\ln 3} \right).$$

For part (b), the logarithms require $x > 1$. Combining them gives

$$(x-1)(x+3) = 5x+7, \quad x^2 - 3x - 10 = 0, \quad (x-5)(x+2) = 0.$$

Only $x = 5$ satisfies $x > 1$. For part (c),

$$3 \ln x + \frac{1}{2} \ln(x+1) - 2x.$$

6. Right-triangle trigonometry.

Let x be an acute angle in a right triangle and suppose that

$$\cos(x) = \frac{2}{3}.$$

- (a) Draw a right triangle that represents this information and label the side adjacent to x and the hypotenuse.
- (b) Use the Pythagorean theorem to find the length of the side opposite x .
- (c) Find the exact values of $\sin(x)$ and $\tan(x)$.

Solution. Since cosine is adjacent over hypotenuse, take the adjacent side to have length 2 and the hypotenuse to have length 3. If the opposite side has length a , then

$$a^2 + 2^2 = 3^2, \quad a^2 = 5, \quad a = \sqrt{5}.$$

Therefore,

$$\sin(x) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\sqrt{5}}{3} \quad \text{and} \quad \tan(x) = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sqrt{5}}{2}.$$

7. Unit-circle trigonometry.

Consider the angle $\theta = \frac{5\pi}{6}$.

- (a) Find the exact coordinates of the point of intersection of the ray that forms the angle θ with the unit circle.
- (b) Find the exact values of $\sin(\theta)$, $\cos(\theta)$, and $\tan(\theta)$.
- (c) Find the angle $\alpha \in [0, 2\pi)$ whose ray intersects the unit circle at the point $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$.

Solution. The unit-circle point corresponding to $\pi/6$ has coordinate magnitudes $\sqrt{3}/2$ and $1/2$. In Quadrant II, the horizontal coordinate is negative and the vertical coordinate is positive. Therefore, the terminal point is

$$(\cos \theta, \sin \theta) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right).$$

Consequently,

$$\sin \theta = \frac{1}{2}, \quad \cos \theta = -\frac{\sqrt{3}}{2}, \quad \tan \theta = \frac{\sin \theta}{\cos \theta} = -\frac{\sqrt{3}}{3}.$$

The point

$$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$$

lies in Quadrant III and has reference angle $\pi/4$. Hence,

$$\alpha = \pi + \frac{\pi}{4} = \frac{5\pi}{4}.$$

8. **Area and perimeter.**

A window consists of a rectangle that is 12 feet tall and 8 feet wide with a semicircle attached along the entire 8-foot top edge. The top edge of the rectangle is the diameter of the semicircle.

- (a) Find the radius of the semicircle.
- (b) Find the total area of the window. Give an exact answer in terms of π .
- (c) Find the outside perimeter of the window. Do not include the shared edge between the rectangle and semicircle.

Solution. The semicircle has diameter 8 feet, so its radius is 4 feet. Its area is half the area of a circle of radius 4. Thus the total area is

$$12(8) + \frac{1}{2}\pi(4)^2 = 96 + 8\pi \text{ ft}^2.$$

The outside perimeter consists of the bottom edge, the two vertical sides, and the semicircular arc. Hence,

$$P = 8 + 2(12) + \frac{1}{2}(2\pi(4)) = 32 + 4\pi \text{ ft.}$$