Math 140: Calculus I (Spring 2026)
Homework 14

Relevant Topics: Riemann sums, Fundamental Theorem of Calculus, substitution, logarithmic, trigonometric, and inverse trigonometric integrals

Instructions. Show enough work to justify your answers.

1. Evaluate the indefinite integral:

$$\int (x+2)\sqrt{x-1}\,dx.$$

2. Evaluate the definite integral:

$$\int_0^{\pi/3} \sin^3(x)\,dx.$$

3. Evaluate the indefinite integral:

$$\int \frac{x}{1+x^4}\,dx.$$

4. Evaluate the definite integral:

$$\int_1^3 \frac{2x}{x^2+1}\,dx.$$

5. Evaluate the indefinite integral:

$$\int \frac{5x+4}{x^2+2x-3}\,dx.$$

6. Evaluate the definite integral:

$$\int_{\pi/6}^{\pi/3} \cot(x)\,dx.$$

7. Evaluate the indefinite integral:

$$\int \frac{1}{\sqrt{4x-x^2}}\,dx.$$

8. Evaluate the definite integral:
$$\int_1^{e^2} \frac{1}{x\sqrt{\ln x}}\, dx.$$

9. Let $F(x)$ be an antiderivative of $f(x)$, and let $g(x)$ be a differentiable function.
   (a) Show that $F(g(x))$ is an antiderivative of $f(g(x))g'(x)$.
   (b) Use this result to explain why the substitution $u = g(x)$ works when evaluating integrals.

10. Consider the definite integral $\int_0^{\pi/2} \sin(x)\cos(x)dx$.
   (a) Sketch the region whose area corresponds to the definite integral.
   (b) USe $u-$substitution to translate this definite integral to a definite integral in u (change the bounds).
   (c) Sketch the region whose area corresponds to the definite integral in u. What is the area of this region?

11. Let $f(x) = F'(x)$ be continuous on $[a, b]$.
   (a) Use the Mean Value Theorem to show that for each subinterval $[x_{i-1}, x_i]$, there exists $c_i \in [x_{i-1}, x_i]$ such that
$$f(c_i)\Delta x = F(x_i) - F(x_{i-1}).$$
   (b) Write the Riemann sum
$$\sum_{i=1}^{n} f(c_i)\Delta x$$
   using this result.
   (c) Explain why the sum telescopes.
   (d) Use this to explain why
$$\int_a^b f(x)\, dx = F(b) - F(a).$$

12. Consider the function $f(x) = \frac{1}{1+x^2}$ on $[a, b]$.
   (a) Show that there exists $c_i \in [x_{i-1}, x_i]$ such that
$$f(c_i)\Delta x = \arctan(x_i) - \arctan(x_{i-1}).$$

(b) Solve for c_i in terms of x_{i-1} and x_i.

(c) Use this to show that the Riemann sum telescopes.

(d) Conclude that

$$\int_a^b \frac{1}{1+x^2}\,dx = \arctan(b) - \arctan(a).$$