Homework 02

Math 140-002: Calculus I (Spring 2026)
Week 2 (Jan 19–Jan 23, 2026)

Relevant topics: Limits (one-sided/two-sided), limit laws, infinite limits, continuity, vertical asymptotes, ϵ–δ definition
Due: Tuesday, Jan 27, 2026.
Instructions: Show your work clearly. Problems 1–6 emphasize computational fluency; Problems 7–12 emphasize concepts and communication.

1. Evaluate $\lim_{x\to 2}(3x-1)$.
2. Evaluate $\lim_{x\to -1}\dfrac{x^2-1}{x+1}$.
3. Evaluate $\lim_{x\to 0}\dfrac{\sin(5x)}{x}$.
4. Evaluate $\lim_{x\to 0}\dfrac{e^x-1}{x}$.
5. Evaluate $\lim_{x\to 1}\dfrac{5x^2+1}{2x^2-3}$.
6. Evaluate $\lim_{x\to \infty}\ln x$.
7. State the ϵ–δ definition of $\lim_{x\to a} f(x) = L$. Then draw a labeled picture that illustrates the definition.
8. Give an example of a function with a removable discontinuity at $x = 1$ and explain why it is removable.
9. Explain why the two-sided limit fails to exist if the one-sided limits are different.
10. For $f(x) = \begin{cases} 2, & x < 1 \\ 5, & x \geq 1 \end{cases}$, find $\lim_{x\to 1^-} f(x)$ and $\lim_{x\to 1^+} f(x)$ and conclude about $\lim_{x\to 1} f(x)$.
11. Consider $g(x) = \frac{1}{(x-2)^2}$. Describe the vertical asymptote and explain what $\lim_{x\to 2} g(x)$ tells you.
12. Let $h(x) = \frac{x^2-1}{x-1}$ for $x \neq 1$. (a) Find $\lim_{x\to 1} h(x)$. (b) Define $h(1)$ so that h is continuous at 1. (c) Define $h(1)$ in a different way so the limit still exists but h is not continuous at 1. Explain the difference.