Homework 01 – Solutions

Math 140-002: Calculus I (Spring 2026)
Week 1 (Jan 12–Jan 16, 2026)

Relevant topics: Functions, inverse functions, trigonometric review, exponential and logarithmic expressions

1. **Problem.** Simplify: $\dfrac{1-\cos^2(x)}{\sin(x)}$ (assume $\sin(x) \neq 0$).

   Solution. By the Pythagorean identity, $1-\cos^2(x) = \sin^2(x)$. Thus

   $$\frac{1-\cos^2(x)}{\sin(x)} = \frac{\sin^2(x)}{\sin(x)} = \sin(x).$$

2. **Problem.** Given $\tan(x) = \dfrac{3}{4}$ with x in Quadrant I, find $\sec(x)$.

   Solution. Using $1+\tan^2(x) = \sec^2(x)$,

   $$\sec^2(x) = 1 + \left(\frac{3}{4}\right)^2 = 1 + \frac{9}{16} = \frac{25}{16},$$

   so $\sec(x) = \frac{5}{4}$ (positive in Quadrant I).

3. **Problem.** Evaluate exactly: $\sin(\arctan(3/4))$.

   Solution. Let $\theta = \arctan(3/4)$ so $\tan\theta = 3/4$. Use a 3-4-5 triangle: $\sin\theta = 3/5$.

4. **Problem.** Evaluate exactly: $\cos(\arcsin(5/13))$.

   Solution. Let $\theta = \arcsin(5/13)$ so $\sin\theta = 5/13$. Then $\cos\theta = \sqrt{1-(5/13)^2} = 12/13$.

5. **Problem.** Solve for x: $\sin(x) = \frac{\sqrt{2}}{2}$ on $[0, 2\pi)$.

   Solution. $x = \frac{\pi}{4}, \frac{3\pi}{4}$.

6. **Problem.** Find all x such that $\cos^2(x) = \dfrac{1}{4}$ on $[0, 2\pi)$.

   Solution. $\cos^2(x) = \frac{1}{4}$ means $\cos(x) = \pm\frac{1}{2}$. On $[0, 2\pi)$ this occurs at

   $$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}.$$

7. **Problem.** Simplify using log rules: $\ln\left(\frac{e^3x^2\sqrt{y}}{(x-1)^5}\right)$ (assume $x > 1$, $y > 0$).

   Solution. $3 + 2\ln x + \frac{1}{2}\ln y - 5\ln(x-1)$.

8. **Problem.** Rewrite as a single logarithm: $2\log_3 x - \frac{1}{2}\log_3 y + \log_3(9)$ (assume $x > 0$, $y > 0$).

   Solution. $\log_3\left(\frac{9x^2}{\sqrt{y}}\right)$.

9. **Problem.** Solve for x: $\ln(x-2) + \ln(x+2) = \ln 15$.

   Solution. $\ln(x^2-4) = \ln 15 \Rightarrow x^2 = 19$. Domain forces $x > 2$, so $x = \sqrt{19}$.

10. **Problem.** Solve for x: $e^{2x} - 5e^x + 6 = 0$.

    Solution. Let $u = e^x > 0$. Then $u^2 - 5u + 6 = 0 \Rightarrow u = 2, 3$, so $x = \ln 2$ or $\ln 3$.

11. **Problem.** Explain (3–5 sentences) why $\arcsin(\sin(x) \neq x$ for all real x, and state precisely when equality holds.

    Solution. arcsin returns values in $[-\pi/2, \pi/2]$, so it inverts sin only on that interval. Equality holds when $x \in [-\pi/2, \pi/2]$ (and then $\arcsin(\sin(x) = x)$.

12. **Problem.** A function satisfies $f(1) = 10$ but $\lim_{x \to 1} f(x) = 3$. Give an explicit example of such a function and briefly verify both conditions.

    Solution. Example: $f(x) = 3$ for $x \neq 1$ and $f(1) = 10$. Then values near 1 are 3 so the limit is 3, while $f(1) = 10$.